



1.

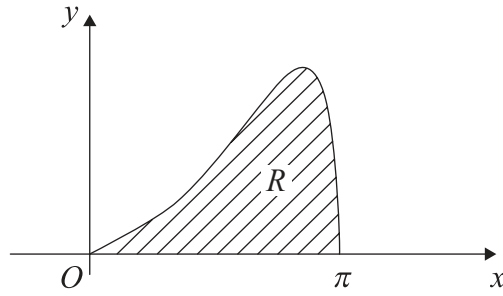


Figure 1

The curve shown in Figure 1 has equation  $y = e^x \sqrt{\sin x}$ ,  $0 \leq x \leq \pi$ . The finite region  $R$  bounded by the curve and the  $x$ -axis is shown shaded in Figure 1.

(a) Complete the table below with the values of  $y$  corresponding to  $x = \frac{\pi}{4}$  and  $\frac{\pi}{2}$ , giving your answers to 5 decimal places.

$x$	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	$\pi$
$y$	0			8.87207	0

(2)

(b) Use the trapezium rule, with all the values in the completed table, to obtain an estimate for the area of the region  $R$ . Give your answer to 4 decimal places.

(4)

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**Question 6 continued**

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Area with horizontal lines for writing answers.







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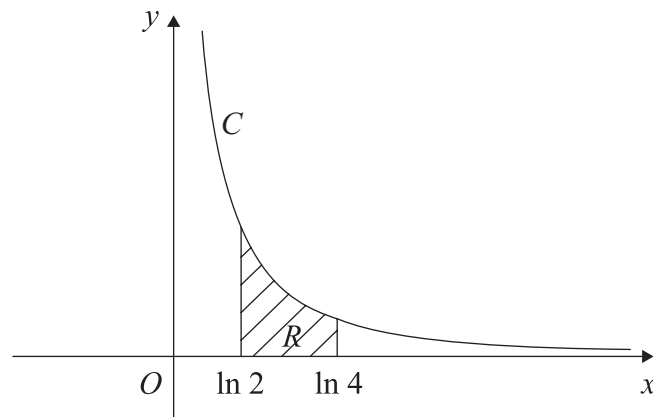


Figure 3

The curve  $C$  has parametric equations

$$x = \ln(t+2), \quad y = \frac{1}{(t+1)}, \quad t > -1.$$

The finite region  $R$  between the curve  $C$  and the  $x$ -axis, bounded by the lines with equations  $x = \ln 2$  and  $x = \ln 4$ , is shown shaded in Figure 3.

(a) Show that the area of  $R$  is given by the integral

$$\int_0^2 \frac{1}{(t+1)(t+2)} dt. \tag{4}$$

(b) Hence find an exact value for this area. (6)

(c) Find a cartesian equation of the curve  $C$ , in the form  $y = f(x)$ . (4)

(d) State the domain of values for  $x$  for this curve. (1)

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8. Liquid is pouring into a large vertical circular cylinder at a constant rate of  $1600 \text{ cm}^3 \text{ s}^{-1}$  and is leaking out of a hole in the base, at a rate proportional to the square root of the height of the liquid already in the cylinder. The area of the circular cross section of the cylinder is  $4000 \text{ cm}^2$ .

(a) Show that at time  $t$  seconds, the height  $h$  cm of liquid in the cylinder satisfies the differential equation

$$\frac{dh}{dt} = 0.4 - k\sqrt{h}, \text{ where } k \text{ is a positive constant.} \quad (3)$$

When  $h = 25$ , water is leaking out of the hole at  $400 \text{ cm}^3 \text{ s}^{-1}$ .

(b) Show that  $k = 0.02$  (1)

(c) Separate the variables of the differential equation

$$\frac{dh}{dt} = 0.4 - 0.02\sqrt{h},$$

to show that the time taken to fill the cylinder from empty to a height of 100 cm is given by

$$\int_0^{100} \frac{50}{20 - \sqrt{h}} dh. \quad (2)$$

Using the substitution  $h = (20 - x)^2$ , or otherwise,

(d) find the exact value of  $\int_0^{100} \frac{50}{20 - \sqrt{h}} dh$ . (6)

(e) Hence find the time taken to fill the cylinder from empty to a height of 100 cm, giving your answer in minutes and seconds to the nearest second. (1)

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